Kinetic Theory of the Dyakonov-Shur Instability

joint work with Andrew Lucas, Stanford University

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Spontaneous generation of terahertz radiation in a 2DEG

Original proposal by Dyakonov and Shur (1993) (implicitly $\tau_{\mathrm{ee}} \ll 1$):

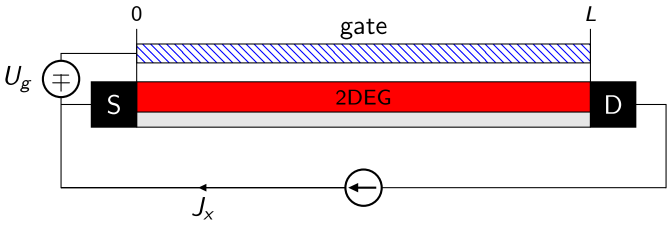

Figure: Schematic experimental setup

Idea: spontaneous fluctuations "blow up" in time: $\propto e^{\gamma t}$ with $\gamma > 0$

Non-standard boundary conditions:

$$\delta n(x = 0) = 0, \qquad \delta J(x = L) = 0$$

This study: transition hydrodynamic ($\tau_{\mathrm{ee}} \ll 1$) to ballistic ($\tau_{\mathrm{ee}} \gg 1$)

Boltzmann kinetic equation

2D Fermi liquid, fermion distribution function $f(\mathbf{x}, \mathbf{p}, t)$

$$\partial_t f + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{x}} f + \mathbf{F}^{\mathrm{ext}} \cdot \nabla_{\mathbf{p}} f = \mathcal{C}[f]$$

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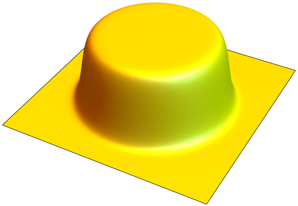
BGK approximation for collision operator:

$$\mathcal{C}[f] = -\frac{f - F_{\text{eq}}[f]}{\tau_{\text{ee}}}$$

with

$$F_{\text{eq}}[f] \equiv \Theta\left(\mu[f] + \mathbf{u}[f] \cdot \mathbf{p} - \epsilon(\mathbf{p})\right),$$

$\mu[f]$ and $\mathbf{u}[f]$ determined self-consistently by density and momentum conservation



Guo et al. 2016, 2017; Jong and Molenkamp 1995; Lucas 2017

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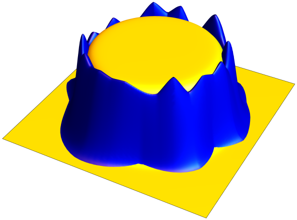

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Boltzmann equation: harmonic representation

$$f(\mathbf{x},\mathbf{p},t) \approx F_{\mathrm{eq}}(\mathbf{p}) + \delta(\epsilon(\mathbf{p})-\mu)\Phi(\mathbf{x},\mathbf{p},t)$$

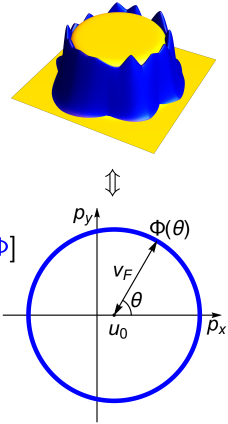
Harmonic representation

$$\Phi(\mathbf{x},\mathbf{p},t) = \mathrm{e}^{-\mathbb{i}\omega t}\sum_{n\in\mathbb{Z}} a_n(\mathbf{x})\,\mathrm{e}^{\mathbb{i}n\theta}$$

Inserted into Boltzmann equation $\rightsquigarrow$

$$\mathbb{i}\omega\Phi = \big(u_0+v_{\mathrm{F}}\cos(\theta)\big)\partial_x\Phi + \frac{2v_{\mathrm{g}}^2}{v_{\mathrm{F}}}\cos(\theta)\partial_x a_0 + \frac{1}{\tau_{\mathrm{ee}}}\mathrm{P}[\Phi]$$

$u_0 = J_x/\rho$	fluid velocity
v_{F}	Fermi velocity
$v_{\mathrm{g}} = \sqrt{\frac{e^2 n_0}{mC}}$	gate "velocity"
$\mathrm{P}[\Phi] = \sum_{\lvert n\rvert\geq 2} a_n\,\mathrm{e}^{\mathbb{i}n\theta}$	collision operator (projection)



Boundary conditions

Dyakonov-Shur boundary conditions:

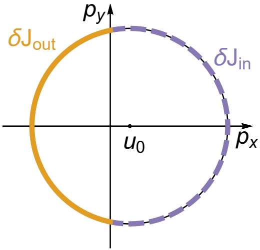
$$\delta n(0) = 0 \quad \Leftrightarrow \quad a_0(0) = 0,$$
$$\delta J(L) = 0 \quad \Leftrightarrow \quad u_0 a_0(L) + \frac{v_\mathrm{F}}{2}(a_1(L) + a_{-1}(L)) = 0$$

Additionally:

"Clean" (specular reflection) boundary conditions:

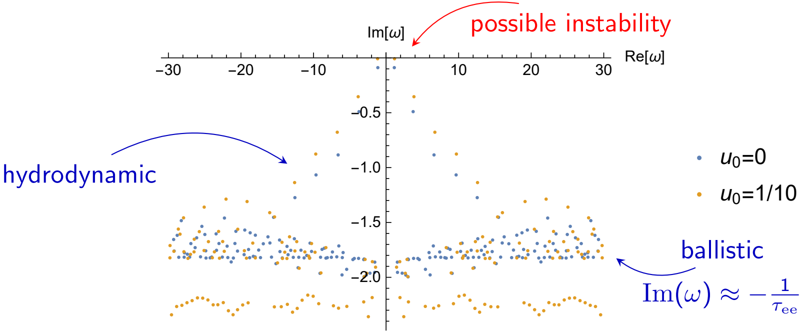
$$\Phi(\theta) = \Phi(\pi - \theta)$$
$$\Updownarrow$$
$$a_n = (-1)^n a_{-n}$$

"Dirty" (uniformly outgoing) boundary conditions:



Eigenvalue spectrum ω (clean BCs)

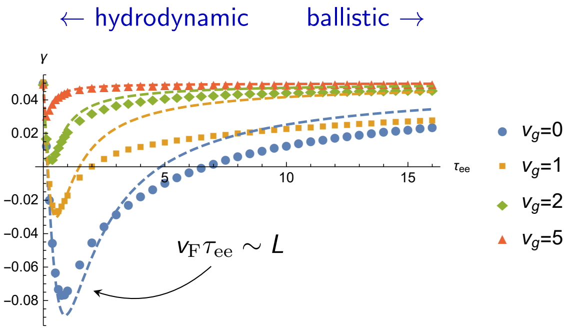
Dyakonov-Shur instability: $\gamma > 0$ with $\gamma = \max(\mathrm{Im}(\omega))$



Parameters: $v_{\mathrm{g}} = 0$, $\tau_{\mathrm{ee}} = 1/2$, clean BCs

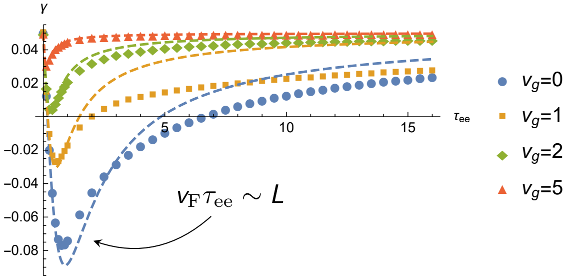
Mendl and Lucas, arXiv:1801.01501 (2018)

γ as a function of τ_{ee} (clean BCs)



γ as a function of τ_{ee} (clean BCs)

← hydrodynamic ballistic →



Dashed: heuristic formula $\color{red}{\gamma} \approx \frac{u_0}{L} - \frac{\pi^2 \nu}{8L^2} \frac{1}{1 + \left(\frac{\pi v_{\mathrm{s}}}{2L}\tau_{\mathrm{ee}}\right)^2}$

$$v_{\mathrm{s}} = \sqrt{\frac{v_{\mathrm{F}}^2}{2} + v_{\mathrm{g}}^2} \quad \text{sound velocity}$$

$$\nu = \frac{v_{\mathrm{F}}^2 \tau_{\mathrm{ee}}}{4} \quad \text{viscosity}$$

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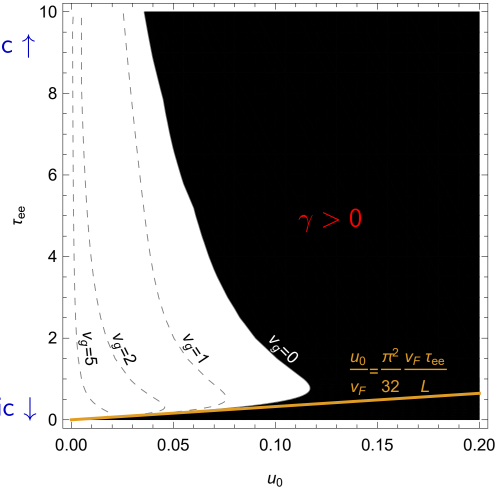
Instability ($\gamma > 0$) depending on τ_{ee} and u_0 (clean BCs)

ballistic ↑

hydrodynamic ↓



Orange: $\gamma = 0$ contour for $\gamma = \dfrac{u_0}{L} - \dfrac{\pi^2 \nu}{8L^2}$

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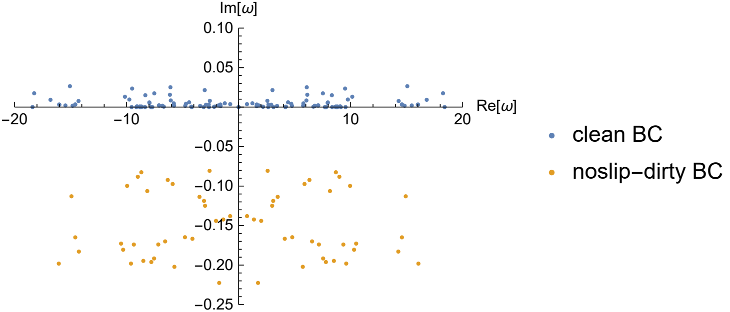
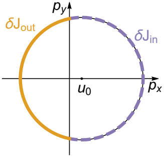
Eigenvalue spectrum in the ballistic limit, clean vs. dirty

Ballistic limit $\tau_{ee} \to \infty$ (no collisions)



Parameters: $u_0 = 1/10$

Dirty boundary conditions:



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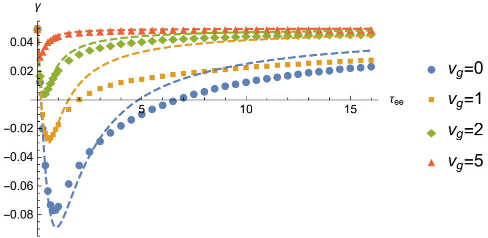
Summary and conclusion

- Fate of the instability in the ballistic limit sensitive to boundary conditions
- Hydrodynamic regime shrinks substantially if $v_\mathrm{g} \gg v_\mathrm{F} \rightsquigarrow$ DS instability need not be hydrodynamic



- Absence of the DS instability in experiments could be used as a heuristic *upper and lower bound* on τ_ee and ν

Mendl and Lucas, arXiv:1801.01501 (2018)

References

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