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Dyakonov-Shur instability across the ballistic-to-hydrodynamic crossover

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We numerically solve semiclassical kinetic equations and compute the growth rate of the Dyakonov-Shur instability of a two-dimensional Fermi liquid in a finite length cavity. When electron-electron scattering is fast, we observe the well-understood hydrodynamic instability and its disappearance due to viscous dissipation. When electron-electron scattering is negligible, we find that the instability re-emerges for certain boundary conditions but not for others. We discuss the implications of these findings for experiments. *Published by AIP Publishing*. https://doi.org/10.1063/1.5022187

The spontaneous generation of terahertz radiation is an important yet challenging problem in applied physics.[1] An interesting proposal is to generate terahertz radiation through the Dyakonov-Shur (DS) instability of a two-dimensional electron gas (2DEG).[2–4] This instability occurs in a uniform flow of current through the 2DEG, subject to non-standard, but experimentally achievable, boundary conditions. In the xy-plane, we consider an infinite strip of 2DEG of width L ($0 \le x \le L$). A uniform, small, background current density $J_x > 0$ is pushed through the strip, and we fix density fluctuations to vanish at $x = 0$ and current fluctuations to vanish at $x = L$. Assuming homogeneity in the y-direction, one finds that for small currents J_x, an instability arises. Spontaneous fluctuations in density and current of amplitude ϵ at time $t = 0$ grow to amplitude $\epsilon e^{\gamma t}$ at time t. In terms of the fluid velocity $u_0 = J_x/\rho$, with ρ being the charge density of the 2DEG

$$\gamma \approx \frac{u_0}{L}. \tag{1}$$

Some signatures of the DS instability have been found in experiment,[5,6] but a clear observation of the DS instability remains challenging. Perhaps one reason is that the original proposal[2] for the instability was in a *hydrodynamic regime*,[7] where electrons collide with other electrons at a rate $1/\tau_{ee}$ much larger than the rate $1/\tau_{imp}$ of electron-impurity/phonon or umklapp collisions. With a few exceptions,[8–12] most electron liquids have *not* been experimentally observed in a hydrodynamic regime. However, an interesting assertion is that the DS instability also exists in a ballistic limit where $\tau_{ee} \to \infty$ and $\tau_{imp} \to \infty$.[13] If the hydrodynamic limit is not necessary, then the DS instability should be observable in a much larger set of 2DEGs and temperature ranges. The difficulty of observing the DS instability would be even more puzzling.

A quick check of this assertion is to compute the viscous correction to γ[2,3,14]

$$\gamma = \frac{u_0}{L} - \frac{\pi^2 \nu}{8L^2}, \tag{2}$$

with $\nu \sim v_F^2 \tau_{ee}$ being the dynamical viscosity and v_F the Fermi velocity. For simplicity in (2), and throughout this letter, we take $\tau_{imp} \to \infty$. The hydrodynamic limit corresponds to $v_F \tau_{ee} \lesssim L$. If this inequality is saturated, we estimate that $\gamma \sim -(v_F - u_0)/v_F \tau_{ee}$, which is expected to be negative. This simple calculation suggests that the DS instability could vanish if electron-electron interactions are weak enough.

In this letter, we explicitly check the fate of the DS instability and numerically calculate γ for a two-dimensional Fermi liquid, using a toy model of (quantum) kinetic theory, with suitable boundary conditions. When $v_F \tau_{ee} \ll L$, we observe quantitative agreement with (2). When $v_F \tau_{ee} \gg L$, we find that the instability becomes somewhat sensitive to boundary conditions. For "clean" boundaries with specular scattering, we numerically find that

$$\gamma \approx \frac{u_0}{L} - \frac{\pi^2 \nu}{8L^2} \frac{1}{1 + \left(\frac{\pi v_s}{2L} \tau_{ee}\right)^2} \tag{3}$$

approximates the instability growth rate. Here, v_s is the speed of sound in the electron fluid. Hence, as $\tau_{ee} \to \infty$, we recover (1), in agreement with Ref. 13. However, for "dirty" boundary conditions with non-specular scattering, we numerically observe that $\gamma < 0$ becomes possible as $\tau_{ee} \to \infty$. Our results demonstrate how boundary conditions on non-hydrodynamic modes could play an important role in suppressing the DS instability in experimental systems.

We now turn to more quantitative details of our study. We compute the low temperature dynamics of an isotropic Fermi liquid in $d = 2$ spatial dimensions, employing the model described in Refs. 8 and 15–17. A thorough introduction to this model is given in the supplementary material (SM); here, we summarize the key points. At low temperatures compared to the Fermi temperature, the most important semiclassical dynamics of a Fermi liquid correspond to the "sloshing" of the Fermi surface itself. If we are only interested in dynamics on length scales large compared to the Fermi wavelength λ_F, then it suffices to solve for the fermion distribution function $f(\mathbf{x}, \mathbf{p})$. Heuristically, f is the "number density of quasiparticles of momentum $\mathbf{p}$ at position $\mathbf{x}$," and

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the Pauli exclusion principle forces $0 \le f \le 1$ for electronic quasiparticles. It is useful to write f as

$$f(\mathbf{x},\mathbf{p}) \approx n_{\mathrm{F}}(\epsilon(\mathbf{p}) - \mu - \Phi(\mathbf{x},\mathbf{p})), \tag{4}$$

with $n_{\mathrm{F}}(x) \approx \Theta(-x)$ at low temperature and Θ the Heaviside step function. If the perturbation Φ is small

$$f(\mathbf{x},\mathbf{p}) \approx f_{\mathrm{eq}}(\mathbf{p}) + \delta(\epsilon(\mathbf{p}) - \mu)\Phi(\mathbf{x},\mathbf{p}). \tag{5}$$

If the Fermi surface is isotropic and (for now) there is no background velocity ($u_0 = 0$), then, the δ function above simply fixes $|\mathbf{p}| = p_{\mathrm{F}}$ and Φ may be parametrized by the angular component θ of $\mathbf{p}$

$$\Phi = \Phi(\mathbf{x},\theta) = \sum_{n\in\mathbb{Z}} a_n(\mathbf{x},t)\mathrm{e}^{\mathrm{i}n\theta}. \tag{6}$$

The harmonic a_0 is proportional to fluctuations in the number density of electrons, while $a_{\pm 1}$ correspond to the local density of $(x \pm \mathrm{i}y)$-momentum. In the toy model described in Refs. 8, and 15–17, the dynamical time evolution of Φ is described by a Boltzmann equation in a relaxation time approximation[18]

$$\partial_t\Phi + v_{\mathrm{F}}\cos(\theta)\partial_x\Phi = -\frac{1}{\tau_{\mathrm{ee}}}\mathrm{P}[\Phi], \tag{7}$$

where

$$\mathrm{P}[\Phi] = \sum_{|n|\ge 2} a_n\mathrm{e}^{\mathrm{i}n\theta}. \tag{8}$$

Due to our setup, we have assumed $\partial_y = 0$. The term on the right hand side of (7) is the linearized collision integral: it relaxes all harmonics of Φ that do not encode a conserved quantity. This model is not microscopically accurate[19,20] but correctly reproduces both $\tau_{\mathrm{ee}} = 0$ and $\tau_{\mathrm{ee}} = \infty$ limits.

Our model does not account for electron-impurity scattering. Heuristically, if τ_{imp} is the electron-impurity scattering rate, then $\gamma \to \gamma - 1/2\tau_{\mathrm{imp}}$.[21,22] High quality 2DEGs can reach $v_{\mathrm{F}}\tau_{\mathrm{imp}} \gtrsim 15\,\mu\mathrm{m}$,[9,23] which is larger than the typical device size.

For mathematical simplicity, we now take

$$\epsilon(\mathbf{p}) = \frac{\mathbf{p}^2}{2m}. \tag{9}$$

To account for background flow, we simply use Galilean invariance: $\partial_t \to \partial_t + u_0\partial_x$ in (7).

In many experimentally realized 2DEGs, the Coulomb interactions are screened by conductors ("gates") a few nm above the sample. This causes an external force[2]

$$\mathbf{F} = \frac{e^2}{C}\nabla n \tag{10}$$

on the electron liquid, analogous to a non-vanishing Landau parameter $\mathcal{F}_0$.[24] Here, C is the capacitance of the gates per unit area and n is the number density of electrons (note $n \propto a_0$). Looking for normal modes where $\Phi \sim \mathrm{e}^{-\mathrm{i}\omega t}$, (7) generalizes to

$$\mathrm{i}\omega\Phi = (u_0 + v_{\mathrm{F}}\cos(\theta))\partial_x\Phi + \frac{2v_{\mathrm{g}}^2}{v_{\mathrm{F}}}\cos(\theta)\partial_x a_0 + \frac{1}{\tau_{\mathrm{ee}}}\mathrm{P}[\Phi], \tag{11}$$

with

$$v_{\mathrm{g}}^2 = \frac{e^2 n_0}{mC}, \tag{12}$$

with n_0 being the background electron density. γ is given by $\max[\mathrm{Im}(\omega_*)]$, where ω_* are the eigenvalues of (11), subject to suitable boundary conditions.

In the hydrodynamic limit, the DS instability is caused by sound waves with dispersion relation

$$\omega \approx (u_0 \pm v_{\mathrm{s}})k - \mathrm{i}\frac{\nu}{2}k^2, \tag{13}$$

with

$$v_{\mathrm{s}} = \sqrt{\frac{v_{\mathrm{F}}^2}{2} + v_{\mathrm{g}}^2}, \quad \nu = \frac{v_{\mathrm{F}}^2\tau_{\mathrm{ee}}}{4}. \tag{14}$$

Neglecting the effects of gating leads to a universal speed of sound $v_{\mathrm{F}}/\sqrt{2}$.[15,16] In the limit where the dominant forces on electrons arise from the gate, $v_{\mathrm{g}} \gg v_{\mathrm{F}}$ and we recover the speed of sound described in Refs. 2 and 13. Assuming $u_0 \ll v_{\mathrm{s}}$, we can estimate the growth rate γ of the instability as follows: The DS boundary conditions amplify sound waves that scatter off of the fixed-current boundary. The rate of these scattering events is $\sim v_{\mathrm{s}}/L$, and the amplification factor is $\sim u_0/v_{\mathrm{s}}$. A sound wave of any amplitude decays at a fixed rate, given in (13), with $k \approx \pi/2L$. Adding the amplification rate and the viscous decay rate leads to (2).

In the ballistic limit, a crude approximation is that the most important corrections to hydrodynamics can be accounted for by a frequency-dependent viscosity[25]

$$\nu(\omega) = \frac{v_{\mathrm{F}}^2\tau_{\mathrm{ee}}}{4(1 - \mathrm{i}\omega\tau_{\mathrm{ee}})}. \tag{15}$$

This equation appears qualitatively consistent with more microscopic calculations in graphene[26] and can be derived by crudely truncating (11) to a few harmonics (see supplementary material). Estimating that we must replace ν in (13) with $\mathrm{Re}[\nu(\omega)]$ and approximating $\omega \approx \pi v_{\mathrm{s}}/2L$ when evaluating $\nu(\omega)$, we obtain our *heuristic* result (3).

When $u_0 = 0$, we can also study the minimal quality factor $Q = \min_k\{-\mathrm{Re}[\omega(k)]/\mathrm{Im}[\omega(k)]\}$ of the waves. Using the approximations of the previous paragraph, we estimate

$$Q \approx \frac{4\tau_{\mathrm{ee}}v_{\mathrm{s}}^2}{\nu} \approx 8 + 16\frac{v_{\mathrm{g}}^2}{v_{\mathrm{F}}^2}. \tag{16}$$

This is in qualitative agreement with the Q-factor reported recently in Ref. 27 in a similar model.

For finite τ_{ee}, we calculate ω^* and γ numerically by truncating (6) to modes with $|n| \le n_{\max}$. The details of the numerical methods can be found in the supplementary material. The DS boundary conditions are

$$0 = a_0(0), \tag{17a}$$

$$0 = u_0 a_0(L) + \frac{v_F}{2}(a_1(L) + a_{-1}(L)). \tag{17b}$$

Choosing the remaining boundary conditions on a_n for $|n| \geq 2$ requires some more care. For example, the number of boundary conditions required by the truncated (11) is $2n_{\max}$ when $u_0 = 0$ and $2n_{\max} + 1$ otherwise. The final boundary condition at $u_0 > 0$ must be chosen so that the $u_0 \to 0$ limit is not singular. This issue is discussed in hydrodynamic language in Ref. 28; our resolution appears to be new. We have found that the proper choice of this boundary condition is $a_2(0) + a_{-2}(0) = 0$. A natural choice to fix the remaining $2n_{\max} - 2$ boundary conditions is to demand that, up to the three prior boundary conditions, $\Phi(\theta) = \Phi(\pi - \theta)$ or $a_n = (-1)^n a_{-n}$. Physically, this boundary condition states that the contacts to the 2DEG are atomically "clean": quasiparticles specularly reflect off of the boundary. Alternative "dirty" boundary conditions are that incoming particles reflect back at a random (outgoing) angle. More details on the choice of boundary conditions are provided in the supplementary material.

For now, let us take clean boundary conditions, up to the caveats of the previous paragraph. We present the entire eigenvalue spectrum in Fig. 1, corresponding to fluctuations which are even under $y \to -y$ (the odd sector decouples). As expected, we observe that the DS instability is carried entirely by sound modes in the hydrodynamic limit, within the full kinetic theory. All non-hydrodynamic degrees of freedom have a finite decay rate: $\mathrm{Im}(\omega_*) \approx -\tau_{ee}^{-1}$. In the infinite volume limit with $u_0 = 0$,[24] it has been shown analytically that $\mathrm{Im}(\omega_*) \approx -\tau_{ee}^{-1}$ for all non-hydrodynamic modes. In all plots in this letter, we work in units where $v_F = L = 1$; thus, $\tau_{ee} < 1$ is "hydrodynamic" and $\tau_{ee} > 1$ is "ballistic."

Continuing to assume clean boundary conditions, we next compute γ as a function of both τ_{ee} and v_g, for fixed $u_0 > 0$; the result is shown in Fig. 2. Regardless of v_g, we find (2) universally in the hydrodynamic limit. Once $v_F\tau_{ee} \sim L$, we observe that γ reaches a *minimal value* $\gamma_{\min}$. For larger τ_{ee}, γ increases as τ_{ee} increases. In fact, we observe that for any v_g, once τ_{ee} is large enough, $\gamma > 0$ (for these boundary conditions). The DS instability occurs in both the hydrodynamic and the ballistic limits, while possibly disappearing at the crossover between them, depending on u_0 and v_g. Figure 2 also confirms that our heuristic estimate (3) captures the qualitative physics of the entire ballistic-to-hydrodynamic crossover. Figure 3 gives an alternate perspective, showing where γ is positive or negative as a function of τ_{ee} and u_0. The "lobe" shape where the instability disappears in Fig. 3 is equivalent to the dip in $\gamma(\tau_{ee})$ observed in Fig. 2, and the DS instability is most suppressed when $v_F\tau_{ee} \sim L$. Although one cannot directly compare the minimal Q-factor in Fig. 2 with (16), as $u_0 > 0$, we do observe that the width and magnitude of the dip in γ both decrease as v_g increases, in agreement with (16). Numerical data in Fig. 2 are qualitatively consistent with a Q-factor $\gtrsim 10$, again in agreement with (16) and Ref. 27.

We have numerically observed that γ is insensitive to boundary conditions in the hydrodynamic limit. The ballistic limit, however, is sensitive to boundary conditions, and an accurate numerical computation of γ can become quite challenging. In the collisionless limit $\tau_{ee} \to \infty$, Eq. (11) for $\Phi(x, \theta)$ decouples at every θ, and a uniform discretization $\theta_j = 2\pi j/n_{\max}$ for $j = 0, 1, \ldots, n_{\max} - 1$ becomes more natural than a (spectral) harmonic truncation: see supplementary material. Thus, the functions $\Phi(x, \theta_j)$ are only coupled via

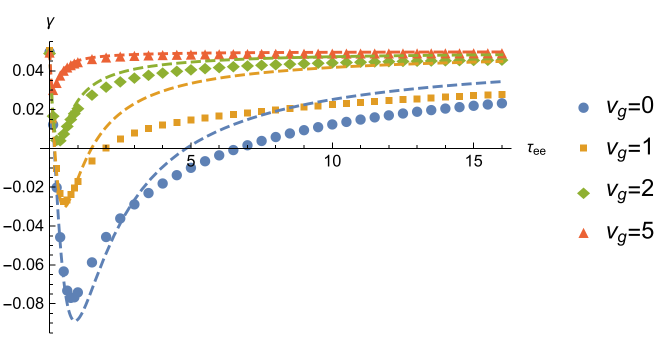

FIG. 1. The even part of the eigenvalue spectrum of (11) with $v_g = 0$, $\tau_{ee} = 1/2$, and DS boundary conditions, for two values of u_0. For small τ_{ee}, the instability arises exclusively in the hydrodynamic sound channel (points on the fictitious curve approaching $\omega = 0$). An infinite number of ballistic modes appears for $\mathrm{Im}(\omega) \approx -1/\tau_{ee}$.

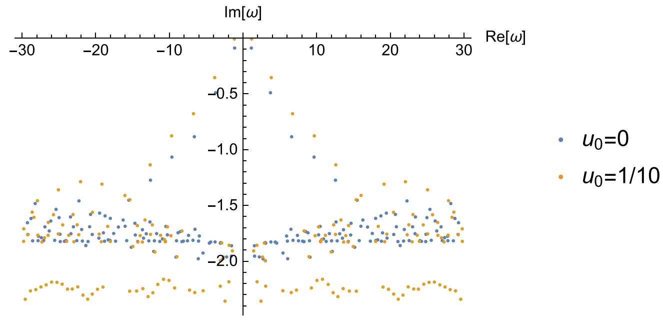

FIG. 2. γ as a function of τ_{ee}, for $u_0 = 1/20$, various gate voltages v_g, and clean boundary conditions. An increasing gate voltage favors the instability. Solid markers show numerical data points, while the dashed line is our heuristic analytical result (3).

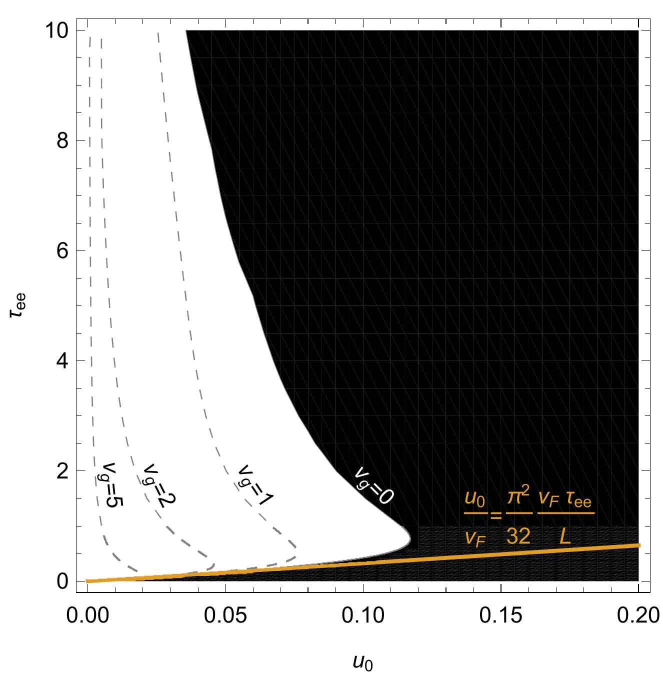

FIG. 3. Values of u_0 and τ_{ee} where $\gamma > 0$ ($\gamma < 0$) are shown in black (white). Dashed lines show the regime of instability at finite v_g. The gold line shows (2).

the boundary conditions. The DS conditions of vanishing density and current fluctuations at $x=0$ and $x=L$ translate to $\sum_j \Phi(0,\theta_j)=0$ and $\sum_j [u_0+v_F\cos(\theta_j)]\Phi(L,\theta_j)=0$, respectively. Besides the DS conditions, we additionally use either clean boundary conditions $\Phi(x,\theta_j)=\Phi(x,\pi-\theta_j)$ at both ends or a "no-slip" reflection $\Phi(0,\theta_j)=\Phi(0,\theta_j+\pi)$ on the left together with "dirty" boundary conditions at $x=L$, such that the distribution $\Phi(L,\theta_j)$ for "outgoing" angles θ_j [i.e., $u_0+v_F\cos(\theta_j)<0$] is uniform. Intuitively, these dirty boundary conditions correspond to an atomically rough contact surface, upon which an incoming quasiparticle is equally likely to be scattered off of the boundary at any scattering angle. A detailed description of the dirty boundary conditions is provided in the supplementary material. Figure 4 shows the numerically computed eigenvalue spectrum of the collisionless kinetic equation for these two variants of boundary conditions, at fixed u_0. We observe that for clean boundary conditions, the instability is present, while for no-slip—dirty boundary conditions, the instability is absent.

Our finding that dirty boundary conditions destroy the DS instability is consistent with Ref. 29, which found that boundary conditions could damp collisionless excitations in a finite length cavity. However, we have also demonstrated the existence of boundary conditions where the DS instability is recovered in the ballistic limit. For certain values of u_0 and v_g, it is possible for the DS instability to persist for arbitrary electron-electron scattering times τ_{ee}, as depicted in Fig. 2.

In this letter, we have numerically computed γ across the ballistic-to-hydrodynamic crossover, in a cavity with the Dyakonov-Shur boundary conditions. We observed that the fate of the instability in the ballistic limit is sensitive to boundary conditions on non-hydrodynamic modes. This provides a further mechanism for suppressing the instability in experimental systems.

The calculations of this paper appear most important for the Fermi liquid of graphene, where $v_F\,\tau_{ee}\sim L$.[10,12] However, it is believed that other 2DEGs, such as GaAs-based heterostructures, are deeper in the hydrodynamic limit, with $\tau_{imp}\gtrsim 10\tau_{ee}$ and $v_s\tau_{ee}\ll L$.[2] However, we observe in Fig. 2 that the hydrodynamic regime (where γ is a decreasing function of τ_{ee}) shrinks substantially if $v_g\gg v_F$; see also Ref. 24. If the modes responsible for the DS instability need not be hydrodynamic even if $\tau_{ee}\ll\tau_{imp}$, then the hydrodynamic assumption frequently employed in the literature may need scrutiny.

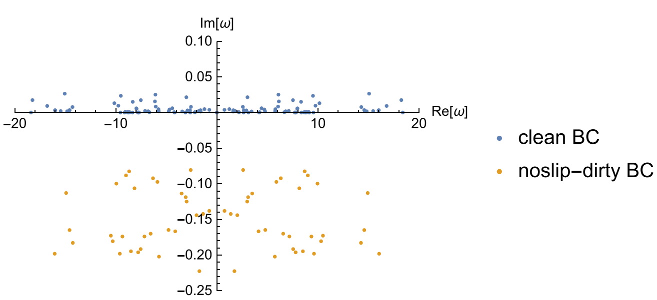

FIG. 4. The eigenvalue spectrum in the ballistic (collisionless) limit $\tau_{ee}\to\infty$, at $u_0=1/10$ and using the θ discretization. Blue dots show the spectrum for clean boundary conditions at both ends and yellow dots for dirty boundary conditions at $x=L$.

We suggest a careful study of electronic boundary conditions in the cavities where the DS instability is searched for, perhaps using transverse electron focusing.[30] This technique has revealed clean boundaries with almost specular reflection in graphene.[30] In a system with clean boundary conditions, our work predicts the DS instability both in a hydrodynamic limit and in a collisionless limit at very low temperatures where electron-phonon scattering is negligible. Furthermore, at higher temperatures, the absence of the DS instability could be used as a heuristic *upper and lower bound* on τ_{ee} and ν. Direct probes of ν are challenging,[31] and indirect measures are imprecise.[32,33] Another measure of τ_{ee} and ν will prove useful for matching theories of electronic hydrodynamics to experiments.

See supplementary material for a detailed derivation of our toy model of quantum kinetic theory, a description of our numerical methods, and a discussion of the boundary conditions employed in our simulations.

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